## Understanding VIX


#### Abstract

In the recent weeks of market turmoil, financial news services have begun routinely reporting the level of the CBOE's Market Volatility Index or "VIX", for short. While this new practice is healthy in the sense that investors are asking for more information in helping to assess the state of the current economic environment and to guide through turbulent waters, it is important to understand exactly what the index means in order to fully capture its usefulness to the market and to avoid misunderstanding and misconception. The purpose of paper is to describe the VIX and its history and purpose, and to explain how it fits within the array of indexes that help describe where the economy stands relative to other points in recent decades.


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## Understanding VIX

In the recent weeks of market turmoil, financial news services have begun reporting the VIX with increasing regularity. This "new" barometer of investor fear-the CBOE's Market Volatility Index-is claimed to have reached unprecedented levels and is now, in fact, causing stock market volatility. ${ }^{1}$ While such misconceptions will, undoubtedly, continue to flow, this article hopes to stem the tide by explaining what VIX is and is not, why it was created, what causes it to move, and why it should be an important piece to investors. Among the lessons to be learned are that the VIX is not new, is not at unprecedented levels, and does not cause market volatility.

## I. The VIX

To begin, VIX is an index, like the Dow Jones Industrial Average (DJIA), computed on a real-time basis throughout each trading day. The only meaningful difference is that it measures volatility and not price. VIX was introduced in 1993 with two purposes in mind. First, it was intended to provide a benchmark of expected short-term market volatility. To facilitate comparisons of the then-current VIX level with historical levels, minute-by-minute values were computed using index option prices dating back to the beginning of January 1986. This was particularly important since documenting the level of market anxiety during the worst stock market crash since the Great Depression-the October 1987 Crash—would provide useful benchmark information in assessing the degree of market turbulence experienced subsequently. Second, VIX was intended to provide an index upon which futures and options contracts on volatility could be written. The social benefits of trading volatility have long been recognized. The Chicago Board Options Exchange (CBOE) launched trading of VIX futures contracts in May 2004 and VIX option contracts in February 2006.

In attempting to understand VIX, it is important to emphasize that it is forward-looking, measuring volatility that the investors expect to see. It is not backward-looking, measuring volatility that has been recently realized, as some commentators sometimes suggest. Conceptually, VIX is like a bond's yield to maturity. Yield to maturity is the discount rate that

[^1]equates a bond's price to the present value of its promised payments. As such, a bond's yield is implied by its current price and represents the expected future return of the bond over its remaining life. In the same manner, VIX is implied by the current prices of S\&P 500 index options and represents expected future market volatility over the next 30 calendar days.

## II. VIX calculation

The VIX was introduced by Whaley (1993). The original index was based on the prices of S\&P 100 (ticker symbol "OEX"), not S\&P 500 (ticker symbol "SPX"), index option prices. The reason was simple. At the time, OEX options were the most actively-traded index options in the U.S., accounting for $75 \%$ of the total index option volume in $1992 .{ }^{2}$ In contrast, the SPX option market was about one-fifth as active, accounting for only $16.1 \%$ of volume. Critical to the timeliness and usefulness of any implied volatility index (of which VIX is only one) is to have it based on prices from a deep and active index option market. In 1992, the dominant source was the OEX option market. A second feature of the original VIX was that it was based on the prices of only eight at-the-money index calls and puts. ${ }^{3}$ Again, this was reasonable. Of the available option series at the time, at-the-money options were by far the most actively traded. Options with exercise prices were away from the current stock index level were less actively traded and frequently had stale price quotes and relatively wide bid/ask spreads. Including such quotes in the real-time computation of the VIX would reduce its timeliness and accuracy.

Over the years since its inception, the structure of index option trading in the U.S. changed in two fundamental ways. First, the SPX option market became the most active index option market in the U.S. Currently, SPX options trade about 12.7 times as frequently as OEX options. ${ }^{4}$ Exactly why the trading volume shifted from one market to the other is unclear. Contributing factors include the fact that the S\&P 500 index is better known, futures contracts on the S\&P 500 are actively traded, and S\&P 500 option contracts are European-style (i.e., exercisable only at expiration), making them easier to value. ${ }^{5}$ On the other side of the coin, the average daily trading volume in OEX options in 2008 was less than half of what it was 16 years earlier. Irrespective of the reason(s), a timely and meaningful implied volatility index requires

[^2]prices from an active underlying index option market. The OEX option market had been supplanted by the SPX option market, and it was time for a change. Second, trading motives of market participants in index option markets changed. In the early 1990s, both index calls and index puts had equally important roles in investor trading strategies. Trading volumes were balanced in 1992, with OEX calls having an average daily trading volume of 120,475 and OEX puts an average daily trading volume of 125,302 . Over the ensuing years, the index option market became dominated by portfolio insurers, who routinely buy out-of-the-money and at-themoney index puts for insurance purposes. During the first ten months of 2008, for example, the average daily volume of SPX puts was 909,748 contracts, over $72 \%$ more than the 525,460 for SPX calls. Indeed, as Bollen and Whaley (2004) show, the demand to buy out-of-the-money and at-the-money SPX puts is a key driver in the movement in SPX implied volatility measures such as VIX. On September 22, 2003, the CBOE changed the VIX calculation to account for both of these fundamental changes in index option market structure. First, they began to use SPX rather than OEX option prices. Second, they began to also include out-of-the-money options in the index computation since out-of-the-money put prices, in particular, contain important information regarding the demands for portfolio insurance and, hence, market volatility. Including additional option series also helps make the VIX less sensitive to any single option price and hence less susceptible to manipulation. ${ }^{6,7}$

It is worth noting that the change from OEX option prices to SPX option prices had little to do with the return/risk properties of the indexes themselves. For all intents and purposes, the S\&P 100 and S\&P 500 index portfolios are perfect substitutes. ${ }^{8}$ Over the period January 1986 through October 2008, the mean daily returns of the S\&P 100 and S\&P 500 were nearly identical, $0.0263 \%$ and $0.0266 \%$, respectively, and the standard deviations of S\&P 100 daily returns was only slightly higher than the S\&P 500 returns, $1.182 \%$ and $1.138 \%$, respectively.

[^3]The correlation between their daily returns was 0.9898 . The near perfect correlation between the return series implies that, holding other factors constant, OEX and SPX options are equally effective from a risk management standpoint. But, other factors are not constant. From the standpoint of maintaining the VIX as a timely and accurate reflection of expected stock market volatility what matters is the depth and liquidity of the index option market. The decision to turn to SPX option prices was warranted.

## The "Investor Fear Gauge"

The VIX has been dubbed the "investor fear gauge." While volatility technically means unexpected moves up or down, the S\&P 500 index option market has become dominated by hedgers who buy index puts when they are concerned about a potential drop in the stock market. Buying insurance is nothing new. People routinely buy fire insurance as a means of insuring their home value in the event of a fire. If the chance of a fire in your neighborhood rises, chances are that your insurance agent will charge more for coverage. The same is true for portfolio insurance. The more investors demand, the higher the price. VIX is an indicator that reflects the price of portfolio insurance.

## III. VIX history

Oftentimes individuals seek to find a precise meaning to an index level. The real benefit from an index, however, comes from comparing its current level to some historical benchmark(s). Consider, for example, the fact that, on October 24, 2008, the Dow closed at 8378.95. Which matters more-the fact that the level equals the sum of the prices of Dow 30 stocks (1052.00) divided by its current divisor (0.1255527090), or the fact that the Dow exceeded 11000 only a month earlier? Most people hone in on the latter and think about its relevance to their portfolio holdings.

Thus, to gauge the normal behavior of the VIX, we should look to its history. ${ }^{9}$ Figure 1 shows week-ending levels of the S\&P 500 and the VIX from the beginning of January 1986

[^4]through October 31, 2008. Several observations are noteworthy. Although not obvious from the weekly figure, the level of VIX reached its record high during the October 19, 1987 market crash. This was the only time VIX ever exceeded a level of 100. By the week's end, VIX fell to a level slightly below 90, however, it continued to persist at abnormally high levels in the ensuing weeks. Another interesting phenomenon shown in the figure is that the VIX frequently spikes upward. Naturally, the market crash in October 1987 is an example, however, the jump in October 1989 is the "mini-crash" resulting from the UAL restructuring failure, the jump in mid1990 occurred when Iraq invaded Kuwait, and the jump in early 1991 corresponds to the attack on Iraq by the United Nations forces. Then two sharp spikes occurred—one in October 1997, and one in October 1998. The October 1997 spike occurred following a stock market sell-off in which the Dow fell 555 points. The October 1998 spike occurred in a period of general nervousness in the stock market. In the aftermath of each spike, the VIX returns to more normal levels. Hopefully, the October 2008 spike is no exception. Finally, although the weekly closing levels of the VIX and the S\&P 500 index appear to spike in opposite directions, there are also times when a run-up in stock prices is accompanied by a run-up in volatility. In January 1999, for example, the VIX was rising (i.e., investors were becoming more nervous) while the level of the S\&P 500 index was rising. The same pattern appears in the first two months of 1995, June and July of 1997, and December 1999. Clearly, investors can become nervous even during market advances.

[^5]Figure 1: Friday closing levels of the S\&P 500 index and the VIX during the period January 3, 1986 through October 31, 2008.


## IV. VIX relation to the stock market

The fact that the VIX spikes during period's of market turmoil is why it has become known as the "investor fear gauge." Two forces are at play. If expected market volatility increases (decreases), investors demand higher (lower) rates of return on stocks, so stock prices fall (rise). This suggests the relation between rate of change in VIX should be proportional to the rate of return on the S\&P 500 index. But, the relation is more complicated. Earlier we argued and documented that increased demand to buy index puts affects the level of VIX. Hence, we should expect to find that the change in VIX rises at a higher absolute rate when the stock market falls than when it rises.

To test this proposition, we regress the daily rate of change of the VIX, $R V I X_{t}$, the rate of change of the S\&P 500 portfolio, $R S P X_{t}$, and the rate of change of the S\&P 500 portfolio conditional on the market going down and 0 otherwise, $R S P X_{t}^{-}$, that is,

$$
R V I X_{t}=\beta_{0}+\beta_{1} R S P X_{t}+\beta_{2} R S P X_{t}^{-}+\varepsilon_{t}
$$

If our proposition is true, the intercept term should not be significantly different from 0 , and the slope coefficients should be significantly less than 0 . As it turns out, our predictions are true. The estimated relation between the rate of change of VIX and the rate of change in SPX is

$$
R V I X_{t}=-0.004-2.990 R S P X_{t}-1.503 R S P X_{t}^{-},
$$

where the number of observations used in the estimation is 5,753 and the regression R -squared is $55.7 \%$. Except for the intercept, all regression coefficients are significantly different from zero at the $1 \%$ level.

The estimated intercept in the regression is -0.004 , and the intercept is not significantly different from 0 . This means that if the SPX does not change over the day, the rate of change in VIX should be negligible. This is not surprising. While the value of stocks is expected to grow through time in order to compensate investors for putting their capital at risk, volatility is not. Volatility tends to follow a mean-reverting process: when VIX is high, it tends to be pulled back down to its long-run mean, and, when VIX is too low, it tends to be pulled back up. The estimated intercept reflects the absence of deterministic growth.

The estimated slope coefficients are both negative and significant, and clearly reflect not only the inverse relation between movements in VIX and movements in the S\&P 500 but also the asymmetry of the movements brought about by portfolio insurance. The way to interpret the coefficients is as follows. If the SPX rises by 100 basis points, the VIX will fall by

$$
R V I X_{t}=-2.990(.01)=-2.99 \%
$$

On the other hand, if the S\&P 500 index falls by 100 basis points, VIX will rise by

$$
R V I X_{t}=-2.990(-.01)-1.503(-.01)=4.493 \%
$$

Because of the demand for portfolio insurance, the relation between rates of change in the VIX and the SPX is asymmetric. VIX is more a barometer of investors' fear of the downside than it is a barometer of investors' excitement (or greed) in a market rally. It is important to note, however, this evidence merely documents correlation and is not intended to express causality.

## V. VIX normal range

Aside from looking at VIX levels, we can attempt to characterize what is normal and abnormal behavior. Table 1 does so in a probabilistic sense. Over its entire history, the median daily closing level of VIX is 18.88. 50\% of time VIX closed between 14.60 and 23.66 (a range of 9.06 points), $75 \%$ of the time VIX closed between 12.04 and 29.14 (a range of 17.10 points), and $95 \%$ of the time VIX closed between 11.30 and 37.22 (a range of 22.92 points). Table 1 also shows a great of variation in what is considered normal from year to year. In 1986, for example, the median daily closing level of VIX was about 19.25. During that same year, the closing levels were about 18.06 and 21.07 about half the time and between 16.92 and 24.24 about $90 \%$ of the time. The widest range experienced is 2008, with the VIX closing between 18.16 and 63.31 (a range of 45.15 index points) about $90 \%$. The second widest range is in 1987-the year of the stock market crash. The 5\% and 95\% percentiles indicate that the range of daily VIX levels was from $16.64 \%$ to 54.11 , or 3,474 basis points.

Table 1: Normal ranges for daily levels of VIX over sample period, January 1986 through October 2008 and by year.

|  | No. of |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | obs. | $5.0 \%$ | $10.0 \%$ | $25.0 \%$ | $50.0 \%$ | $75.0 \%$ | $90.0 \%$ | $95.0 \%$ |
| All | 5,754 | 11.30 | 12.04 | 14.60 | 18.88 | 23.66 | 29.14 | 34.22 |
| 1986 | 252 | 16.92 | 17.34 | 18.06 | 19.25 | 21.07 | 23.64 | 24.24 |
| 1987 | 253 | 16.64 | 17.28 | 20.85 | 22.66 | 26.81 | 46.25 | 54.11 |
| 1988 | 253 | 17.44 | 18.11 | 20.35 | 24.06 | 27.21 | 34.20 | 36.16 |
| 1989 | 252 | 15.51 | 15.90 | 16.47 | 17.30 | 18.22 | 20.60 | 22.59 |
| 1990 | 253 | 16.55 | 17.32 | 18.31 | 21.16 | 26.11 | 28.96 | 30.49 |
| 1991 | 251 | 14.99 | 15.29 | 16.02 | 17.29 | 19.13 | 21.87 | 24.46 |
| 1992 | 254 | 12.18 | 12.73 | 13.36 | 14.76 | 15.98 | 17.33 | 17.96 |
| 1993 | 251 | 10.43 | 10.92 | 11.40 | 12.27 | 13.03 | 14.05 | 14.38 |
| 1994 | 252 | 10.26 | 10.49 | 11.29 | 12.80 | 14.47 | 15.63 | 16.07 |
| 1995 | 252 | 10.71 | 11.00 | 11.51 | 12.29 | 13.18 | 13.80 | 14.15 |
| 1996 | 254 | 13.43 | 14.70 | 15.72 | 16.78 | 18.16 | 19.39 | 20.45 |
| 1997 | 253 | 19.92 | 20.17 | 21.11 | 22.20 | 24.64 | 27.80 | 30.36 |
| 1998 | 252 | 18.06 | 18.82 | 20.43 | 22.61 | 27.67 | 36.37 | 41.49 |
| 1999 | 252 | 19.70 | 20.73 | 22.39 | 24.29 | 26.59 | 28.73 | 30.34 |
| 2000 | 252 | 19.67 | 20.76 | 22.45 | 24.89 | 27.61 | 30.19 | 31.50 |
| 2001 | 248 | 21.80 | 22.37 | 23.85 | 26.24 | 30.64 | 34.20 | 36.34 |
| 2002 | 250 | 19.79 | 20.84 | 22.44 | 29.19 | 35.31 | 41.25 | 43.89 |
| 2003 | 252 | 16.45 | 16.78 | 18.96 | 21.21 | 26.92 | 34.77 | 35.80 |


| 2004 | 252 | 12.63 | 13.05 | 14.28 | 15.32 | 16.55 | 18.13 | 18.91 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2005 | 252 | 10.75 | 11.08 | 11.66 | 12.52 | 13.64 | 14.83 | 15.58 |
| 2006 | 251 | 10.52 | 10.78 | 11.35 | 12.00 | 13.60 | 16.18 | 17.73 |
| 2007 | 251 | 10.34 | 10.97 | 13.11 | 16.33 | 21.65 | 25.24 | 26.48 |
| 2008 | 212 | 18.16 | 19.45 | 21.14 | 23.79 | 27.55 | 45.24 | 63.31 |

An important way of judging market anxiety is to examine the persistence with which VIX remains above certain extraordinary levels. From Table 1, we know that the chance of observing a VIX level above 34.22 is $5 \%$. Suppose we re-examine the VIX history to count the number of consecutive days that VIX has remained above a level of 34.22. Four periods last more than 20 days can be identified: October 16 through December 22, 1987 (47 days), August 28 through October 31, 2002 (46 days), September 26 through October 31, 2008 (26 days so far), and January 8 through February 8, 1988 (22 days). So, yes, we are experiencing abnormal behavior, but, no, it is not unprecedented. We just tend to forget.

## VI. VIX prediction about future volatility

Unlike the DJIA, which only has meaning relative to its history, the VIX also has a simple, probabilistic interpretation concerning the expected range of the rate of return on the S\&P 500 index level over the next 30 days. Figure 2 shown below provides a "quick-and-ready" method for interpreting the level of VIX. ${ }^{10}$ To understand how to use the figure, assume that the current level of VIX is, say, 60. Reading up from 60 on the horizontal axis to the line labeled " $50 \%$ " and then across to the vertical axis, we see that the expected range of S\&P 500 returns over the next 30 days is about $11.5 \%$. This means that, if VIX is at 60 , chances are $50-50$ that the rate of return on the S\&P 500 index will be up or down by less than or more than $11.5 \%$ over the next 30 days. The lines labeled $75 \%$ and $95 \%$ offer different levels of probability for a given VIX level. At a VIX level of 60, the $75 \%$ (95\%) line indicates that expected range of S\&P 500 returns over the next 30 days is about $20 \%$ (34\%). In other words, if the VIX is at 60, the chances that the S\&P will go up or down by less than $20 \%$ ( $34 \%$ ) over the next 30 days are $75 \%$ (95\%). Conversely, at a VIX of 60, the chances that the S\&P will go up or down by more than $20 \%$ (34\%) over the next 30 days are $25 \%$ (5\%). The figure is intended to be a quick reference

[^6]guide for interpreting VIX. For those who want to be more precise, the lines are generated using the following relations: ${ }^{11}$

Expected range at $50 \%=0.1947 \times V I X$
Expected range at $75 \%=0.3321 \times V I X$
Expected range at $95 \%=0.5658 \times V I X$
At a VIX level of 60, the exact values for the expected ranges are $11.68 \%$, 19.92\%, and $33.95 \%$, respectively.

Figure 2: Expected range of S\&P 500 returns over the next 30 days conditional on current VIX level.


## VII. VIX performance as a predictor

An obvious question to ask about the prediction rule described in the last section is "How well does it perform?" To answer this question, a simple experiment was performed. At the beginning of each of the 274 month in the sample period, the level of VIX was recorded. Based

[^7]on the level of VIX at the beginning of the month, $50 \%, 75 \%$, and $95 \%$ expected ranges were computed using the above formulas. The rate of return of the S\&P 500 over the month was then computed, and the number of times the return fell outside the range during the 274 months of the sample period was recorded. Of the 274 trials, 95 or $34.7 \%$ fell outside the $50 \%$ range, 20 or $7.3 \%$ fell outside the $75 \%$ range, and 3 or $1.1 \%$ fell outside the $95 \%$ range. In other words, VIX works reasonably well as a predictor of the expected of stock index movements. ${ }^{12}$

## VIII. Creation of other volatility indexes

VIX is not unique as a stock market volatility index. It is merely the first to have been introduced and has a first-mover advantage. The CBOE (2003) methodology for computing the index is not unique to the prices of S\&P 500 index options. It can be applied to any index option market. Indeed, the CBOE has already applied the methodology to create a volatility index for the NASDAQ 100 (the "VXN") and for the DJIA (the "VXD"). The only important requisite is that the underlying index option market has deep and active trading across a broad range of exercise prices. Not surprisingly, VIX has also attracted imitators internationally. The NYSE Euronext has applied the CBOE (2003) methodology to index options listed on the AEX (an index of 25 stocks traded in Amsterdam), the BEL20 (an index of 20 Belgium stocks), the CAC40 (an index of 40 French stocks), and the FTSE 100 (an index of 100 stocks traded in the United Kingdom). Examining the co-movements of the volatility indexes in different countries will undoubtedly be a subject of future research and discussion.

## IX. Summary

VIX is a forward-looking index of the expected return volatility of the S\&P 500 index over the next 30 days. It is implied from the prices of S\&P 500 index options, which are predominantly used by the market as a means of insuring the value of their stock portfolios. While the current levels of VIX are high by historical standards, they are not the highest they have ever been nor have they been as persistently high during at least two episodes previously.

[^8]
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[^1]:    ${ }^{1}$ A couple of recent quotes include "Now we're in uncharted territory..." in "Volatility: Measure Could Signal a Bottom" (Philadelphia Inquirer, November 1, 2008) and "The VIX is a self-fulfilling prophecy..." in "On Wall Street, Eyes Turn to the Fear Index" (New York Time, October 20, 2008).

[^2]:    ${ }^{2}$ See Whaley (1993, p.72).
    ${ }^{3}$ The details of the original index computation are provided in Whaley (1993, pp. 80-82).
    ${ }^{4}$ This figure is based on the first ten months of trading in 2008.
    ${ }^{5}$ OEX options, on the other hand, are American-style and can be exercised at any time during the option's life.

[^3]:    ${ }^{6}$ From the perspective of trading derivatives contracts on the VIX, the change in methodology also provided a means of trading VIX by passively using SPX option contracts. This provides market makers in VIX futures and options with a less expensive means of hedging their inventory and promotes narrower bid/ask spreads in the VIX futures and options markets. Under the original version of VIX, dynamic hedging was required. See Carr and Madan (1998) and Demeterfi, Derman, Kamal, and Zou (1999).
    ${ }^{7}$ The details of the revised VIX computation are provided in CBOE (2003). An Excel-based spreadsheet for computing the VIX is provided in Whaley (2006, pp. 553-562).
    ${ }^{8}$ Both the S\&P 100 and the S\&P 500 are market capitalization-weighted stock indexes. As of October 31, 2008, all S\&P 100 stocks are contained within the S\&P 500 index portfolio and account for $62.46 \%$ of the S\&P 500's total market capitalization. The 34 highest market cap stocks in the S\&P 500 are also the 34 highest market cap stocks in the S\&P 100. Of the 100 highest market cap stocks in the S\&P 500, 70 are from the S\&P 100.

[^4]:    ${ }^{9}$ The CBOE changed the composition of VIX on September 22, 2003. For the period, January 2, 1986 through September 19, 2003, it was based on S\&P 100 index option prices. Afterward, it was based on S\&P 500 index option prices. Earlier we showed that the S\&P 100 and S\&P 500 index portfolios were virtually perfect substitutes for one another, so using the VIX history based on S\&P 100 prices until September 22, 2003 (i.e., the cleaner, more accurate historical series) and then the VIX history based on SPX option prices is a sensible way to develop a full VIX history. Another, perhaps more accurate, approach is to attempt to address the scale difference in the volatilities of the two volatility series. Earlier we showed that the daily standard deviation of the S\&P 500 index was 0.011378

[^5]:    while the daily standard deviation of the S\&P 100 index was 0.011822 . The fact that the S\&P 500 volatility is only $96.24 \%$ of the S\&P 100 volatility reflects the fact that the S\&P 500 has marginally lower risk. Consequently, we adjust the pre-September 22, 2003 levels of the OEX option-based VIX to reflect this scale difference. To estimate the adjustment, we regress the daily VIX based on SPX options prices on the daily VIX based on OEX option prices during the period September 22, 2003 through October 31, 2008, suppressing the intercept term in the regression. The estimated slope coefficient is 0.9727 . To create the SPX option-based VIX prior to September 22, 2003, we simply scale the OEX option based VIX by a factor of 0.9727 . With or without the adjustment, the qualitative interpretations of the VIX history are unaffected.

[^6]:    ${ }^{10}$ The figure makes two simplifying assumptions: (a) the rate of return on the S\&P 500 over the next 30 days is normally distributed, and (b) the expected rate of return on the S\&P 500 over the next 30 days is zero. Neither assumption is unreasonable.

[^7]:    ${ }^{11}$ These relations are derived from the cumulative standard normal density function. A random number drawn from a unit normal distribution has a $50 \%$ chance of being within 0.6745 standard deviations of 0 , a $75 \%$ chance of being within 1.1504 standard deviations of 0 , and a $95 \%$ chance of being within 1.9600 standard deviations from 0 . Since VIX is an annualized standard deviation, we must scale each of the coefficients by the square root of 12 to convert them to monthly volatilities (e.g., $0.6745 / \sqrt{12}=0.1947$ ).

[^8]:    ${ }^{12}$ If anything, the range appears to wide relative given the empirical results. One possible explanation for this result is that investors pay more than the actuarial value for their portfolio insurance.

